Electromagnetic Theory: PHAS3201, Winter 2008
6. Reflection and Refraction at a Plane Dielectric Surface

1 Refractive Index

Origin

- Why do materials have a refractive index ?
- We have *stated* that n = c/v_p
- In our solution for plane waves, we considered only vacuum
- We will consider two cases with media:
  1. A non-conducting dielectric (ϵ_r)
  2. A conducting system (briefly)
- Refractive index comes directly from Maxwell's equations

TAKE NOTES

Phase velocity

- We see, finally:

$$k^2 = \hat{\epsilon}\frac{\omega^2}{c^2} \tag{1}$$

- But the phase velocity, $v_p = \omega/k = c/\sqrt{\hat{\epsilon}}$
- What about the two cases ?
- A dielectric simply has $n = \sqrt{\epsilon_r}$
- A conducting system has a *complex* dielectric constant and refractive index
- So the refractive index comes *directly* from the dielectric constant

2 Reflection & Refraction

Geometry

- We have incident, reflected and refracted waves
- $\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0 \exp i\left(\mathbf{k}\cdot\mathbf{r} - \omega t\right)$
- $\mathbf{E}'(\mathbf{r}, t) = \mathbf{E}'_0 \exp i\left(\mathbf{k}'\cdot\mathbf{r} - \omega t\right)$
- $\mathbf{E}''(\mathbf{r}, t) = \mathbf{E}''_0 \exp i\left(\mathbf{k}''\cdot\mathbf{r} - \omega t\right)$
- Phases in prefactors

TAKE NOTES

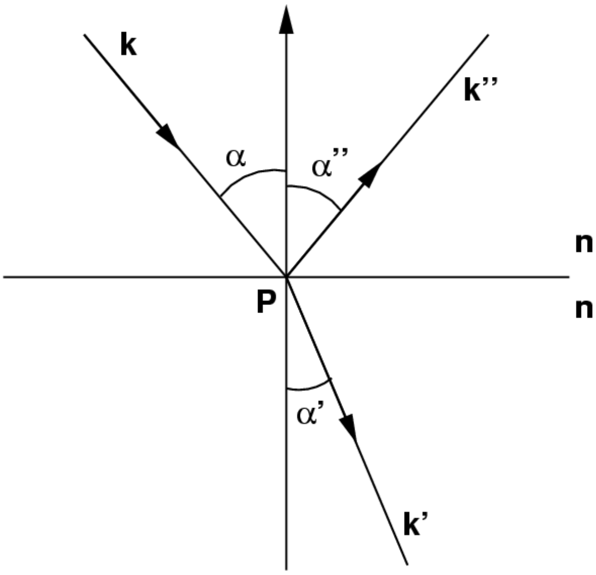

Figure 1: Wave with wavevector k incident at point P travelling from medium with refractive index n to medium with refractive index n′

Two Laws

- This enables us to write:
$$\alpha = \alpha'' \tag{2}$$
- Angle of incidence equals angle of reflection
- $k \sin\alpha = k' \sin\alpha'$
- But $k_n = n\omega/c$, so
$$n' \sin\alpha' = n \sin\alpha \tag{3}$$
- Snell's Law

Changes of Amplitude

- What happens to the energy of a reflected and refracted wave ?
- We consider perfect plane waves, with infinite extent
- By using boundary conditions on the fields, we will follow the *amplitudes*
- Two important new quantities:

$$r = \frac{|\mathbf{E}''|}{|\mathbf{E}|} \tag{4}$$
$$t = \frac{|\mathbf{E}'|}{|\mathbf{E}|} \tag{5}$$

TAKE NOTES

Fresnel Relations

- These are called the Fresnel Relations

$$r_{\|} = \frac{n' \cos\alpha - n\cos\alpha'}{n'\cos\alpha + n\cos\alpha'} \quad (6)$$

$$r_{\perp} = \frac{n \cos\alpha - n'\cos\alpha'}{n\cos\alpha + n'\cos\alpha'} \quad (7)$$

$$t_{\|} = \frac{2n\cos\alpha}{n'\cos\alpha + n\cos\alpha'} \quad (8)$$

$$t_{\perp} = \frac{2n\cos\alpha}{n\cos\alpha + n'\cos\alpha'} \quad (9)$$

- They tell us about *amplitudes* of waves
- For power (or intensity) we need their square

TAKE NOTES

3 Special Angles

There are two particularly important angles where interesting things happen to the reflection and transmission coefficients.

Brewster Angle

- At some point, $r_{\|} \to 0$ but $r_{\perp} \neq 0$
- This is the Brewster angle
- All the power of the incident $\mathbf{E}_{\|}$ wave goes into refracted wave
- But in general the incident wave will have a $\mathbf{E}_{\perp}$ component
- Reflected light will be *polarised* perpendicular to the plane of incidence

TAKE NOTES

Brewster Angle(2)

- The final result is:

$$\alpha_B = \tan^{-1}\frac{n'}{n} \quad (10)$$

- Many shiny dielectrics (paint, wet roads etc) have $n'/n_{air} \sim 1.5$
- So $\alpha_B = 50 - 60°$
- Notice that $r_{\|}$ changes sign, so direction of reflected vectors changes

Critical Angle

- Is there a situation where the transmission goes to zero ?
- The trivial solution is $\alpha = \pi/2$
- This explains glancing reflection from still lakes and glass
- If $n > n'$, Snell's law gives:

$$\sin\alpha' = \frac{n}{n'}\sin\alpha \quad (11)$$

- There will be some angle α above which $\sin\alpha' > 1$, which is unphysical
- We define *critical* angle as $\alpha_C = \sin^{-1}(n'/n)$

TAKE NOTES

Total Internal Reflection

- Consider an air/glass interface
- Air has $n = 1$
- Glass has $n \simeq 1.5$
- What are the Brewster angles from each side of the interface ?
- What about the critical angles ?
- Plot intensities versus angle

Intensities

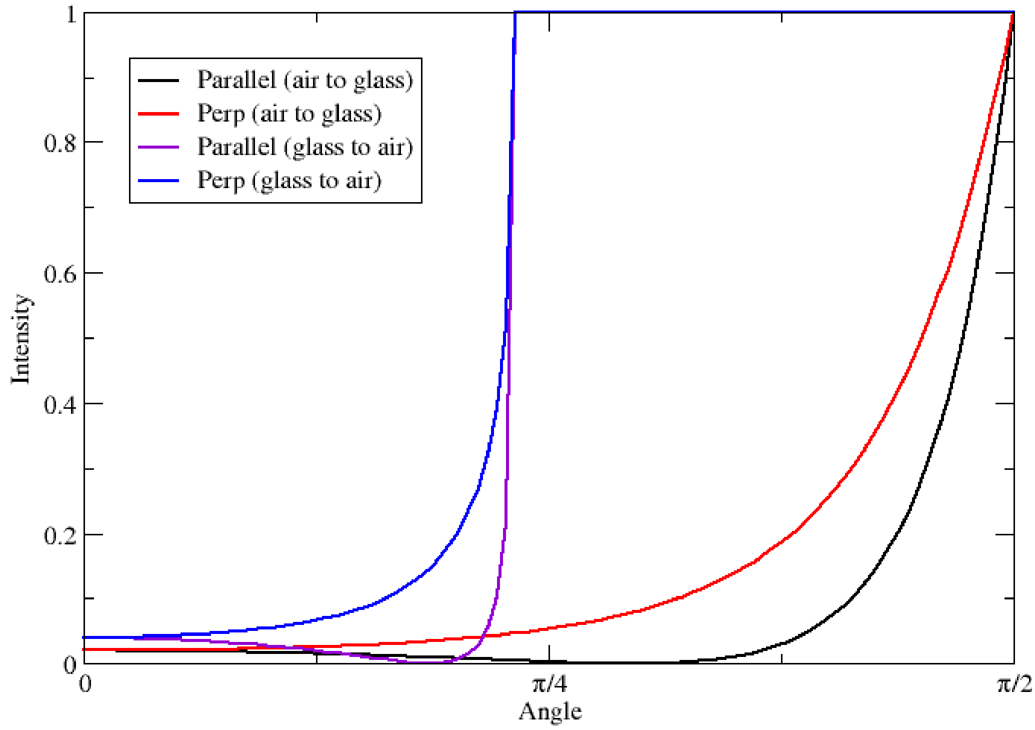

Figure 2: Intensities of reflected EM waves from air/glass interface (in both directions) as a function of angle, for components parallel and perpendicular to plane of incidence.

TAKE NOTES

Evanescent Waves

- We write for the electric field below the interface:

$$\mathbf{E}' = \mathbf{E}'_0 \exp\left(-k' S z\right) \exp i \left(k' \left(n/n'\right) \sin \alpha' x - \omega t\right) \tag{12}$$

- This is a travelling wave along x which decays exponentially with z

- It is called an *evanescent* wave
- If another piece of material is brought up below the interface, a new wave can be excited, driven by the evanescent wave