Electromagnetic Theory: PHAS3201, Winter 2008
Problem Sheet 2

Hand in your answers by **Friday 31 October**, *either at the lectures or Dr Bowler's pigeonhole in Physics. Marks per section are shown in square brackets.*

1. (a) Starting from the Coulomb gauge ($\boldsymbol{\nabla} \cdot \mathbf{A} = 0$), show that the Ampere's law for *static* fields gives:

$$\nabla^2 \mathbf{A}(\mathbf{r}) = -\mu_0 \mathbf{J}(\mathbf{r}) \tag{1}$$

[2]

   (b) How can this be simplified for a current density along $\hat{\mathbf{k}}$ in a homogeneous medium ? Assume that the current density is a function of x and y only. [1]

   (c) What is the equation for the *scalar* potential in a dielectric material of permittivity ϵ with an external charge density $\rho_{\rm ex}$? Compare your answers for the scalar and vector potentials, and comment on the direction of the resulting $\mathbf{E}$ and $\mathbf{B}$ fields. [2]

   (d) The boundary conditions on $\mathbf{B}$ at an interface between two media with a current density $\mathbf{K}$ can be written $\mathbf{B}_1 - \mathbf{B}_2 = \mu_0 (\mathbf{K} \times \mathbf{n})$, where $\mathbf{n}$ is the normal to the interface. The boundary conditions on $\mathbf{A}$ are $\mathbf{A}_1 = \mathbf{A}_2$ and:

$$\frac{\partial \mathbf{A}_1}{\partial n} - \frac{\partial \mathbf{A}_2}{\partial n} = -\mu_0 \mathbf{K} \tag{2}$$

   Prove equation (2). [Hint: set $\mathbf{n}$ along z, $\mathbf{K}$ along x and write out the components of $\mathbf{B}$. Note that $\mathbf{A}$ depends on $\mathbf{K}$.] [5]

2. (a) Find $\mathbf{H}$ for a cylinder of radius a lying along and centred on the z-axis, which carries a uniform current density $\mathbf{J} = j\mathbf{k}$ over its area, both inside and outside the cylinder. Use *cartesian* coordinates (N.B. $\hat{\phi} = y\mathbf{i} - x\mathbf{j}$) [3]

   (b) Now find $\mathbf{H}$ for a cylinder of radius b lying along the z-axis and centred on $(c, 0)$, which carries a uniform current density $\mathbf{J} = -j\mathbf{k}$ over its area, both inside and outside the cylinder. Again, use *cartesian* coordinates. [Hint: shift the origin to find the field, then shift back] [3]

   (c) Hence find $\mathbf{H}$ *outside* a cylinder of radius a with a cylindrical hole, radius b, centred on $(c, 0)$. [Hint: Remember that electromagnetism is linear, so a hole is equivalent to superposing opposite currents.] [4]

3. (a) Consider a neutral piece of dielectric material with permittivity ϵ in a field $\mathbf{E}$ with the cross-section shown in Fig. 2(a). Find the surface polarisation charge density:

   (i) On the end faces, using the relation between $\mathbf{E}$ and $\mathbf{P}$ and the definition of $\sigma_{\rm P}$ [3]

   (ii) On the sloping face, using the charge on the flat face and the charge neutrality of the system [2]

   (b) Calculate the magnetic dipole moment for the shape in Fig. 2(b) with current I flowing. [5]

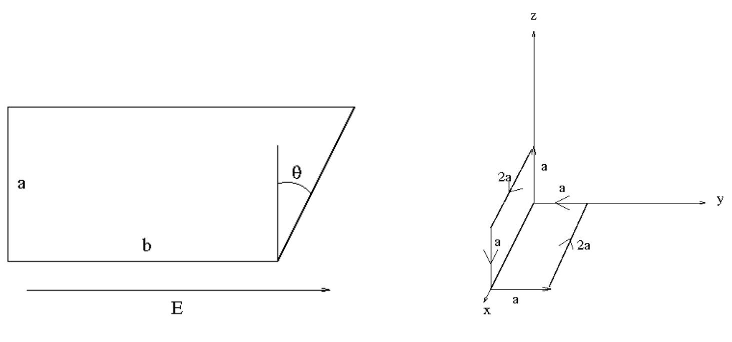

Figure 2: (a) Cross-section of dielectric materials (b) current loop