Electromagnetic Theory: PHAS3201, Winter 2008
Problem Sheet 1

Hand in your answers by **Friday 17 October**, *either at the lectures or Dr Bowler's pigeonhole in Physics. Marks per section are shown in square brackets.*

1. (a) Consider the function $f(x, y, z) = 2x^2y + zx$. Doing *all differentation* explicitly, find:

   (i) ∇f [2]

   (ii) $\nabla \cdot (\nabla f)$ [2]

   (iii) $\nabla \times (\nabla f)$ [2]

   (b) Consider the two fields in Cartesian coordinates:

$$\begin{aligned} \mathbf{E}_1 &= (yz, xz, yx) \\ \mathbf{E}_2 &= (xy, yz, xz) \end{aligned}$$

   Find $\nabla \cdot \mathbf{E}_1, \nabla \times \mathbf{E}_1, \nabla \cdot \mathbf{E}_2, \nabla \times \mathbf{E}_2$ explicitly. [4]

2. (a) Use Gauss' Law to find the field $\mathbf{E}(\mathbf{r})$ due to a point charge q at the origin. Be careful to justify the direction of the field and *all* steps of the argument. [2]

   (b) Consider a sphere of dielectric material with radius R, permittivity ϵ and a point charge q embedded at the centre of the sphere.

   (i) Use Gauss' Law to find the electric displacement $\mathbf{D}(\mathbf{r})$ for all values of $\mathbf{r}$ [2]

   (ii) Write down $\mathbf{E}$ and hence $\mathbf{P}$, defining any symbols you use [2]

   (iii) Use the divergence *in spherical polar coordinates* to write down the volume polarisation charge density and $\nabla \cdot \mathbf{E}$. Where does the solution break down, and why ? [2]

   (iv) What is the surface polarisation charge density ? [2]

3. (a) Consider the vector field $\mathbf{v} = 6\mathbf{i} + yz^2\mathbf{j} + (3y + z)\mathbf{k}$, and the contour C shown in Fig. 1(a) below.

   (i) Perform the line integral $\oint_C \mathbf{v} \cdot d\mathbf{l}$ [3]

   (ii) Check your answer using Stokes' theorem [3]

   (b) Calculate the total flux of the electric field through the face of the cube shaded in Fig. 1(b). [Hint: This is done most easily using symmetry: create a symmetric system where the charge is at the *centre*; find the total flux using Gauss' Law; then work out the flux for the portion of the solution relevant here. The "obvious" solution involving a double integral is rather complicated.] [4]

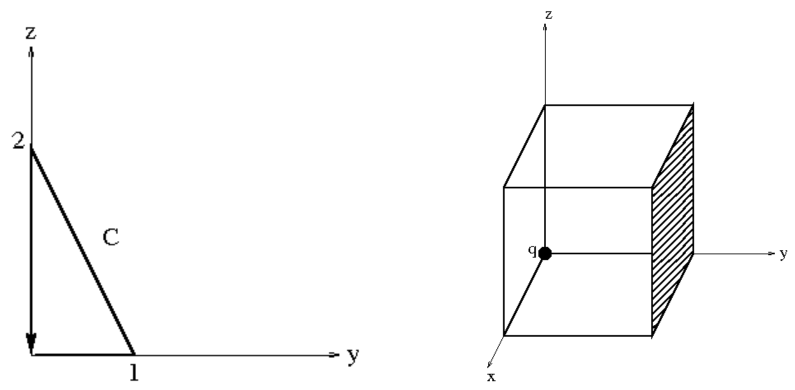

Figure 1: (a) Contour for **v**; (b) Cube with charge at corner.