Electromagnetic Theory: PHAS3201, Winter 2008
4. Ferromagnetism

Ferromagnetism represents the earliest discovery of a phenomenon which results from quantum phenomena: lodestones were used in navigation by the Phoenicians several thousand years ago, while the detailed understanding of ferromagnetism was not worked out until 1928 (by Heisenberg). We will cover the details at a qualitative level only.

1 Atomic-level Picture

We start by considering the effect of the unpaired electron in the 3d shell.

Intrinsic Moments

- There are *intrinsic* moments at the atomic level
- Unpaired electron spins give the direction of the moments
- There is a strong short range force between neighbouring atoms
- The atoms will align in the lowest energy configuration

TAKE NOTES

FM Orientations

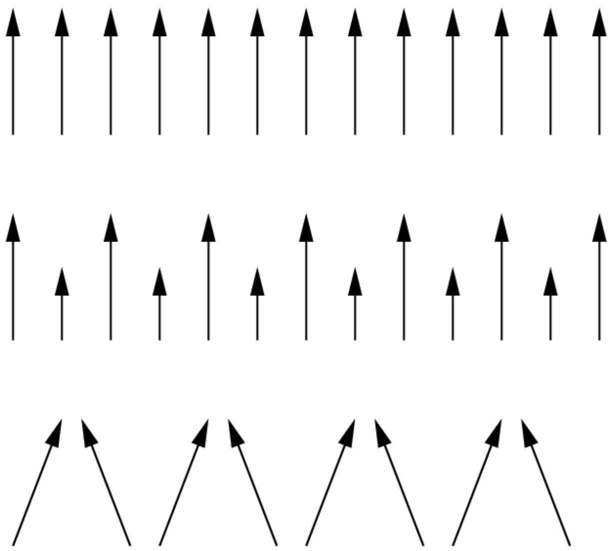

Figure 1: Examples of ferromagnetic ordering

- Ferromagnetic ordering can take different forms
- The defining characteristic is a local, parallel ordering
- Ordering depends on temperature
- Ordering may only be local

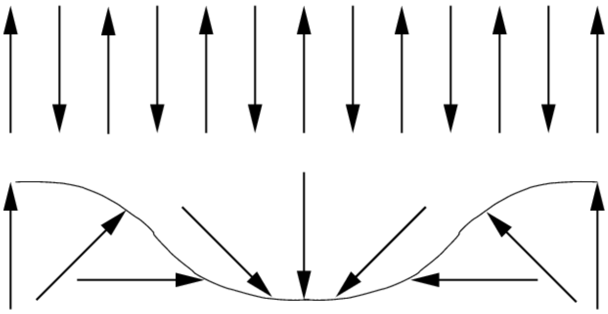

Figure 2: Examples of antiferromagnetic ordering

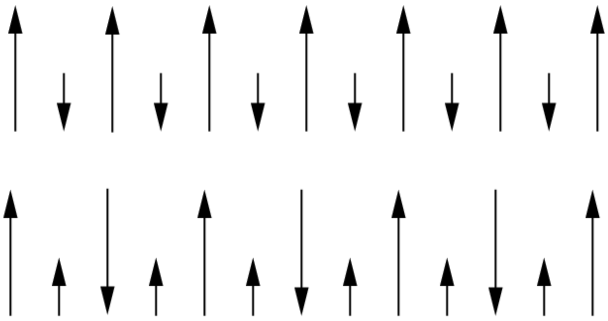

Figure 3: Examples of ferrimagnetic ordering

Other Orientations

- Anti-ferromagnetic ordering has anti-parallel local ordering
- Ferrimagnetic ordering shows both spin components but a net moment
- Also known as ferrite materials
- Important materials (more later)

TAKE NOTES

Domains

- When cooled with no external field, the domains are disordered
- With an external field, they align
- The resulting magnetisation is large (strong moments)
- $\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M})$ gives $\mathbf{B} \gg \mathbf{H}$
- Ferromagnetism amplifies magnetic effects strongly

2 B & H: Macroscopic Effects

If we want to investigate magnetic properties of different materials, it's useful to remember that $\mathbf{H}$ arises from *free* currents only (i.e. those flowing in wires or coils), so that we can always impose a value of $\mathbf{H}$ on any sample (particularly a ferromagnetic one). The resulting induction $\mathbf{B}$ will depend on $\mathbf{H}$ and $\mathbf{M}$. As we change $\mathbf{H}$, the magnetisation will change and we can detect the results using Faraday's law to detect changes in $\mathbf{B}$(we will discuss a circuit for this later).

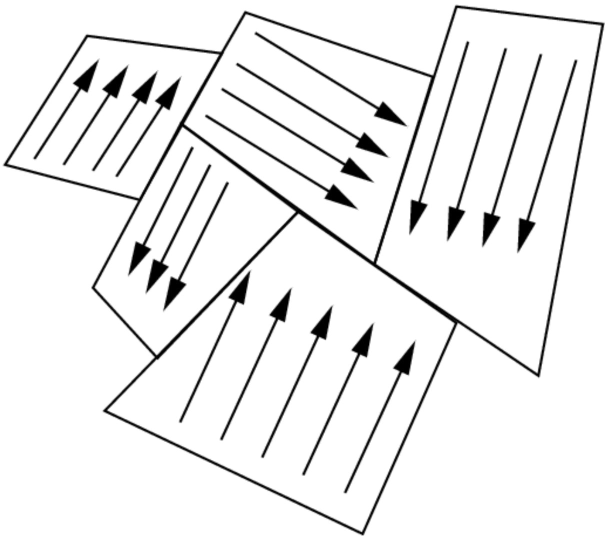

Figure 4: Ferromagnetic domains

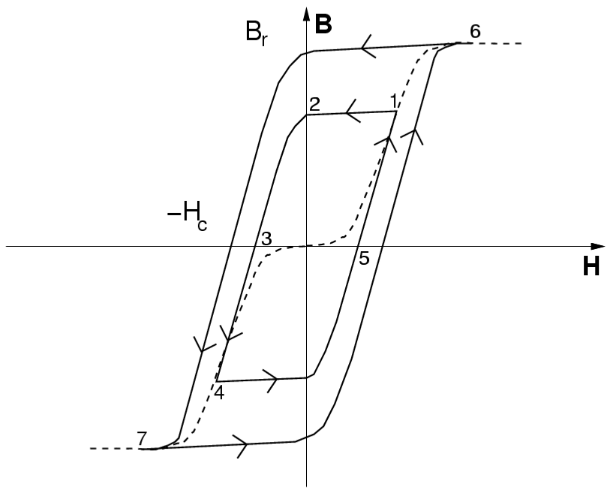

Figure 5: B-H curves for a ferromagnetic material

Hysteresis
TAKE NOTES

Definitions

- Saturation magnetisation: value of M when domains are fully aligned
- Saturation intensity, H_s: magnetic intensity required to produce saturation
- Saturation induction, B_s: magnetic induction at saturation
- Remanence, B_r: value of $\mathbf{B}$ on the major loop when $\mathbf{H}$ is *returned* to zero
- Coercivity, H_c: value of $\mathbf{H}$ required to reduce $\mathbf{B}$ to zero after saturation
- Effective relative permeability, $\mu_{\mathrm{r-eff}}$: maximum value of $B/\mu_0 H$

Be careful with $\mu_{\mathrm{r-eff}}$: it is (sometimes) loosely defined as "the point where a straight line from the origin is tangent to the B/H curve". There is also the maximum differential permeability, taken as the maximum *slope* of the B-H curve. $\mu_{\mathrm{r-eff}}$ can also be referred to as K_{max}, with $K = \mu/\mu_0$.

Soft	$\mu_{\mathrm{r-eff}}$	H_c (A/m)	B_s (T)
3% Si-Fe	4.0×10^4	8.0	2.0
Mn-Zn ferrite	1.5×10^3	0.8	0.2
Mumetal	1.0×10^5	4.0	0.6
Supermalloy	1.0×10^6	0.2	0.8
Hard		H_c (A/m)	B_r (T)
5% Cr steel		5.0×10^3	0.94
Alnico		8.0×10^4	0.62
Co_5Sm		1.0×10^6	1.50
Fe-Nd-B		1.0×10^6	1.30

Table 1: Table of properties of ferromagnetic materials

Real B-H curve

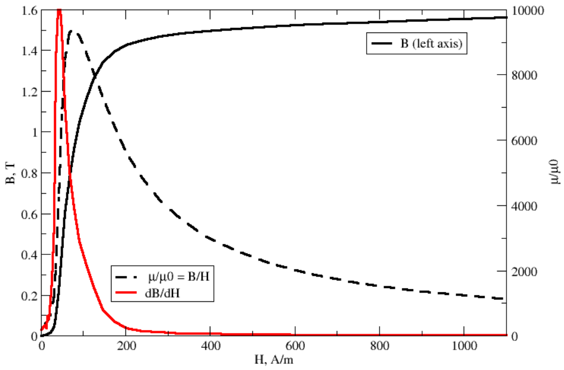

Figure 6: Measured B-H curve for a thin steel sample, with $\mu/\mu_0 (= B/H)$ and dB/dH calculated from the data

The B-H curve for steel (Fig. 6) also shows the curve B/H (which would be μ/μ_0 if the material were linear) and the differential, dB/dH. For the normal magnetisation curve, people often use the definition $\mu(\mathbf{H}) = \mathbf{B}/\mathbf{H}$ *despite* the fact that the relationship is non-linear in a ferromagnet.

Properties

TAKE NOTES

More Properties

3 Examples

We will now consider some simple examples of electromagnetic systems, and applications of coils to generate $\mathbf{H}$ fields: the solenoid, the bar magnet, the electromagnet (combining the two), the toroidal electromagnet and the fluxmeter.

Solenoid

- Tightly wound coil carrying current I
- N turns, length L
- We will calculate the $\mathbf{B}$ field from vector potential

TAKE NOTES

Ferromagnets	Curie T (K)	$\mu_0 M_s$ (T)
Fe	1043	$\sim$2
Co	1388	$\sim$1.6
Ni	627	$\sim$0.6
Gd	293	1.98
Dy	85	3.0
Ferrimagnets	Curie T (K)	$\mu_0 M_s$ (T)
Fe_3O_4	858	0.51
$CoFe_2O_4$	793	0.475
Antiferromagnets	Neel T (K)	
MnO	122	
FeO	198	
NiO	600	
$MnCl_2$	2	

Table 2: Table of critical temperatures and saturation magnetisation for ferro-, antiferro- and ferrimagnetic materials

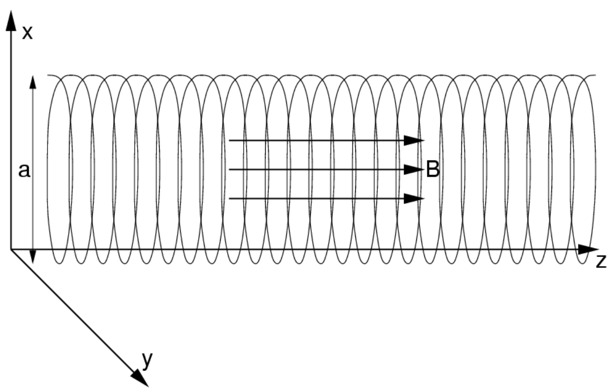

Figure 7: Geometry of a solenoid

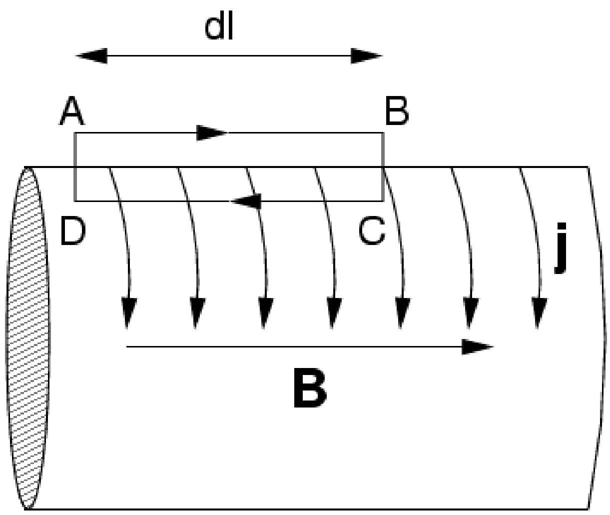

Figure 8: Geometry of a bar magnet

Key Results

- Far from the ends, field is axial. Remember that $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$
- But $\mathbf{J} = 0$ *inside* the solenoid
- We can show that this gives $\hat{\mathbf{x}} \frac{\partial B_z}{\partial y} - \hat{\mathbf{y}} \frac{\partial B_z}{\partial x} = 0$
- This is only obeyed if the field is *uniform*
- The field can be found to be $B_z = \mu_0 IN/L$ from Ampere's law

Bar Magnet

- Assume *uniform* magnetisation, $\mathbf{M} = (0, 0, M_z)$
- There will be an associated surface magnetization current, $\mathbf{j}_m$
- This will be $\mathbf{j}_m = (0, M_z, 0)$ in cylindrical polar coordinates
- Compare this with $\mathbf{j}_f = NI/L$ in the solenoid (*free* current)

Bar Magnet Field

- We can use the same geometry for the solenoid and the bar magnet
- Apply Ampere's law around the loop ABCD
- $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{loop}$

  TAKE NOTES

Magnetic Field

- We find:

$$B_z dl = \mu_0 j dl \tag{1}$$

- For the solenoid, $j = NI/L$, $B_z = \mu_0 NI/L$
- For the bar magnet, $j = M_z$, $B_z = \mu_0 M$

- For the solenoid, $M = 0$ so $\mathbf{H} = \mathbf{B}/\mu_0$
- For the bar magnet, $\mathbf{M} = \mathbf{B}/\mu_0$, so $\mathbf{H} = 0$
- We would get this result using boundary conditions on $\mathbf{H}$
- Combining the two gives an electromagnet, with $j = j_f + j_m$
- We find $B_z = \mu_0 \left(NI/L + M_z \right)$ but $H_z = NI/L$

Toroid

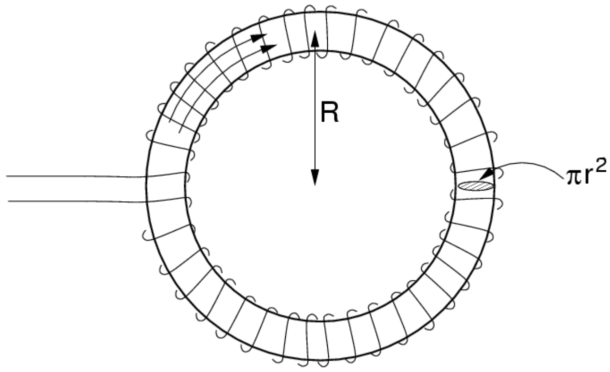

Figure 9: Geometry of a toroidal electromagnet

- A toroidal, closed FM loop
- Closed lines of $\mathbf{B}$
- Assume radius of ring $R \gg r$, x-section radius
- N turns total, current I

TAKE NOTES

Fluxmeter

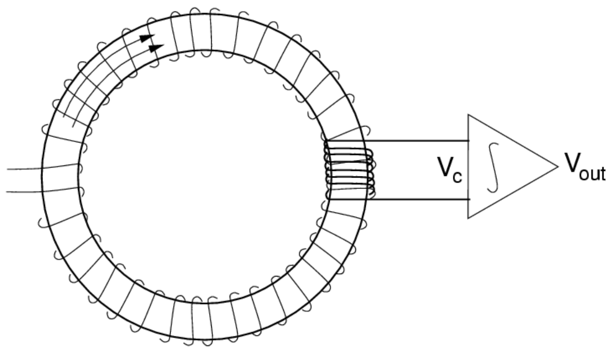

Figure 10: Fluxmeter

- Wind an extra coil, with n_c turns, over the magnetising coil
- Connect to a fluxmeter (op-amp circuit with low impedance R_c)
- $V_{out} = K \int_0^t I_c dt$

TAKE NOTES

Fluxmeter

- So we have:

$$V_{out}(t) = C\Delta B(t) \tag{2}$$

- with C a measurable constant
- We *impose* $\mathbf{H}$ via current, toroidal loop
- We *measure* $\mathbf{B}$ via fluxmeter output
- This provides *direct* evidence of B, H
- Plot hysteresis loops etc

4 Energy Density

Here we think about the magnetic equivalent of the energy density in the electric field. Consider a general circuit with resistance R in a magnetic field. Then $V + \mathcal{E} = IR$, with $\mathcal{E}$ the induced EMF due to the magnetic field.

Energy in circuit

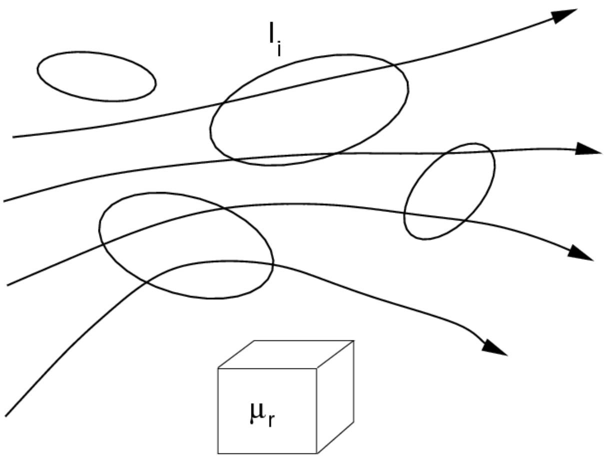

Figure 11: Collection of circuits and magnetic media

- Work done moving $dq = Idt$ is:

$$Vdq = VIdt = -\mathcal{E}Idt + I^2Rdt \tag{3}$$

- If we ignore Ohmic losses (I^2R), $dW_b = Id\Phi$
- This is the energy required to maintain the current I

TAKE NOTES

Energy Density in a Solenoid

- We have the total energy, $W = \frac{1}{2}\sum_i I_i\Phi_i$
- Consider each turn as a circuit: $\Phi_i = \Phi = \pi r^2 B$, $\sum_i I_i = NI$
- But $NI = Hl$ and $V = \pi r^2 l$, so $W = \frac{1}{2}HBV$

- The energy *density* is:

$$U = \frac{1}{2}HB \tag{4}$$

- More generally, $U = \frac{1}{2}\mathbf{H} \cdot \mathbf{B}$

TAKE NOTES

Summary of Linear Media

- Linear: χ is independent of $\mathbf{E}$ (or χ_m of $\mathbf{B}$)
- Isotropic: $\mathbf{P}$ is parallel to $\mathbf{E}$ (or $\mathbf{M}$ to $\mathbf{H}$)
- $\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P}$
- $\mathbf{P} = \chi\mathbf{E}$ so $\mathbf{D} = \epsilon\mathbf{E}$, with $\epsilon = \epsilon_0\left(1 + \chi/\epsilon_0\right)$
- $\boldsymbol{\nabla} \cdot \mathbf{D} = \rho_f$
- $\mathbf{H} = \mathbf{B}/\mu_0 - \mathbf{M}$
- $\mathbf{M} = \chi_m\mathbf{H}$ so $\mathbf{B} = \mu_0\mu_r\mathbf{H}$ with $\mu_r = 1 + \chi_m$
- $\boldsymbol{\nabla} \times \mathbf{H} = \mathbf{J}_f$

Summary of Non-Linear Media

- Unpaired electrons give *intrinsic* moment
- There is a *short-range* force which aligns these spins
- If parallel, *ferromagnetic* ordering
- If anti-parallel, *anti-ferromagnetic* ordering
- Local domains of aligned atoms form (up to microns across)
- Long-range forces arrange these opposed to each other
- Highly non-linear B vs. H curves: *hysteresis*
- Energy density, $U = \frac{1}{2}\mathbf{B} \cdot \mathbf{H}$