10.1098/rsta.2003.1213

THE ROYAL SOCIETY

Quantum computing in the solid state: the challenge of decoherence

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Published online 2 June 2003

The current status of solid-state implementations of quantum computing is briefly described. There are numerous candidate proposals, but only comparatively recently have some of them begun to progress to the point of demonstrating coherent motion of the individual quantum bits (qubits), and the controlled coupling of more than one qubit remains a significant challenge. We also present a generalization of the fluctuation–dissipation theorem that provides a relationship between the coherent evolution entangling two spatially separated qubits, and the associated irreducible decoherence. This relationship may be used to bound the maximum attainable figures of merit in proposed two-qubit gates.

Keywords: quantum computing; decoherence; master equations

1. Introduction

At first sight it might appear crazy for solid-state physicists to enter the field of quantum information processing (QIP). After all, the entities with which they deal are typically much more strongly interacting than the atomic components used in atom-trap or ion-trap approaches (see elsewhere in this issue; for example, Wineland *et al.* 2003; Gulde *et al.* 2003; Porto *et al.* 2003; García-Ripoll & Cirac 2003), and this would seem to make them much less suitable for the coherent manipulation of quantum information. But this has not prevented the appearance of a number of very imaginative proposals for using the numerous excitations in condensed matter systems for QIP.

At the moment these are just proposals. But very significant progress is being made towards surmounting the technological challenges required to turn the proposals into reality. In this paper we will start by putting some of these proposals in the context of the well-known 'DiVincenzo checklist' of criteria that a successful quantum computer would have to satisfy. Our survey will by no means be comprehensive, but it will serve to illustrate something of the tremendous range of possible condensed-matter approaches to QIP. We will end this survey by discussing a relatively new class of condensed-phase approaches, which may have a chance of overcoming some of the apparent difficulties with other methods.

One contribution of 20 to a Discussion Meeting 'Practical realizations of quantum information processing'.

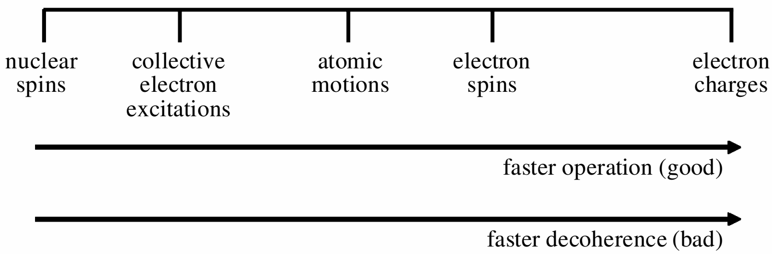

Figure 1. Schematic of the range of some of the types of state available for representing QIP in condensed matter, and the corresponding time-scales.

We will then move on to consider a fundamental relationship between the rate at which entanglement can be created between two qubits, and the amount of decoherence that is introduced in the process of accomplishing this. We will find that as far as the figure of merit of a two-qubit gate is concerned, the most important parameter is the frequency spectrum of the environment, rather than the absolute strength of the qubit-environment coupling. While this relationship applies to potential quantum gates in many different contexts, it may impose particular limitations on condensed-phase systems.

2. The DiVincenzo criteria and some solid-state qubits

David DiVincenzo's (1996) list was an attempt to enumerate the tests that any proposal for performing QIP would have to pass before it could be considered credible. The requirements are

(i) a well-defined Hilbert space within which to represent quantum information (typically in the form of a number of two-level qubits);

(ii) a means of preparing pure states within this set;

(iii) a means of performing arbitrary unitary transformations on the state of the qubits (typically by implementing arbitrary single-qubit operations and at least one non-trivial two-qubit operation) while

(iv) avoiding decoherence for long enough to compute; and, finally,

(v) some means of reading out the state of the qubits at the end of the calculation.

One striking point about a solid is that the large number of particles involved, and the richness of the excitation spectrum, means that there are a very large number of possible ways of representing and manipulating the quantum information. If we add to this the control over the growth and structure of materials now available, the variety of possible ways of representing the quantum information might seem overwhelming. One way of classifying the range of systems involved is to consider the range of time-scales available for the manipulation of quantum information, corresponding to the range of excitation energies ΔE and corresponding time-scales $\hbar/\Delta E$. These time-scales range over roughly 12 orders of magnitude, from the time-scales of milliseconds associated with nuclear spin precession to the femtoseconds associated with interband electronic excitations in semiconductors; a schematic of this range is shown in figure 1.

(*a*) *Collective excitations: qubits in superconductors*

This field has been very clearly reviewed by Wendin (2003). Possible superconducting qubits include

(i) a 'Cooper pair box', which can be placed in a superposition of occupation-number states (Nakamura *et al.* 1999, 2002; Vion *et al.* 2002);

(ii) a superconducting ring containing a weak link, in which either the magnetic flux or the charge state is used for the qubit state (Mooij *et al.* 1999).

Considerable progress has been made in isolating these systems from the environment, demonstrating that coherent evolution of the states is possible (Nakamura *et al.* 1999; van der Wal *et al.* 2000), and in performing elementary single-qubit quantum operations.

(*b*) *Qubits based on nuclear spin*

The very first experiments demonstrating QIP were performed using liquid-state nuclear magnetic resonance (NMR) (Cory *et al.* 1997; Jones 2000), and the state of the art is described by Jones (2003). The impossibility of preparing a pure initial state in a liquid at any accessible temperature, and the extreme difficulty of single-spin readout, means that the operations have to be performed with a large ensemble of spins and a weakly perturbed thermal density matrix. It is therefore natural to consider working at ultra-low temperatures in a solid and localizing the qubits so each can be individually addressed. The best developed proposal of this type is due to Kane (1998); this envisages depositing ^{31}P nuclei near the surface of an isotopically pure ^{28}Si crystal. The quantum information would then reside in the nuclear spins, but would be manipulated by a combination of an external magnetic field and the hyperfine interaction with the shallow donor electron state associated with the P impurity. In order to control this hyperfine coupling, and the exchange coupling between the electrons on neighbouring donors, gate electrodes have to be grown on the surface in well-defined positions above and between the impurity atoms. The combined technological challenges associated with the fabrication of the device are formidable, but considerable progress has been made with the controlled deposition of the P atoms (see Oberbeck *et al.* (2002) and the review by Clark *et al.* (2003)); coherent quantum operation of the qubits remains to be demonstrated.

(*c*) *Qubits based on electron spin*

An alternative qubit is the electron spin; since the Bohr magneton is much larger than the nuclear magneton ($\mu_B/\mu_N = m_p/m_e$) electron spins can be manipulated in electron paramagnetic resonance experiments much more rapidly than nuclei in NMR. Correspondingly, however, the decoherence times are also faster (typically of the order of microseconds rather than milliseconds). One of the first such proposals (Loss & DiVincenzo 1998) involved using electron spins at quantum dots as the qubits, and coupling them by the effective exchange interaction resulting from inter-dot electron tunnelling, which might be controlled by external electrodes as in the Kane proposal; since the dots are more easily located than individual P impurities, one might expect the fabrication challenges to be slightly less severe in this case.

Another family of proposals would use the electron spins in molecular species for the qubits; one particularly promising candidate appears to be the endohedral fullerene species N@C_{60}; the $S = 3/2$ spin state associated with the N atom has an exceptionally long relaxation time ($T_1 \approx 1$ ms, $T_2 \approx 0.01$ ms at room temperature) in the free state, thanks to its near-perfect isolation from the environment by the fullerene 'cage' (Almeida Murphy *et al.* 1996). These spins could potentially be coupled by controlling the behaviour of nearby electron states via appropriate nanostructures, as in the Kane proposal.

A different set of approaches involves mobile, rather than bound, electrons; for example, single electrons can be trapped within lithographically defined semiconductor quantum wires by the potential wells of a surface acoustic wave (SAW), and there is the possibility of performing two-qubit operations by bringing the wires close enough together to allow exchange to occur (Barnes *et al.* 2000). This scheme has the particular advantage that a natural single-spin readout mechanism suggests itself: when the SAW discharges the electrons into a hole gas, recombination occurs to produce photons whose polarizations map the spins of the original electrons (Barnes *et al.* 2000).

(*d*) *Qubits based on interband electron excitations*

The largest energy scales, and hence most rapid responses, normally accessible in solids involve interband transitions, for example, in the production of excitons. One possible candidate qubit is an excitation of a single quantum dot; coherent evolution has been demonstrated (Bonadeo *et al.* 1998) between two different polarization states of a quantum dot, and there are a number of methods proposed to use such excitonic transitions as the basis for quantum computations. For example, one could use the presence or absence of an exciton in a given dot to correspond to the qubit's $|0\rangle$ and $|1\rangle$ states, and produce inter-qubit coupling by the Förster non-local exchange interaction (Quiroga & Johnson 1999). The challenge in operating such systems is their extremely fast response; the Rabi time for an exciton in a dot may be only femtoseconds (Bonadeo *et al.* 98), while the dephasing time may be as rapid as $T_2 \approx 40$ ps in some structures.

(*e*) *Optical control of inter-qubit interactions*

One particularly interesting recent development is the idea that the interactions between solid-state qubits might themselves be controlled optically. This has the advantage that the positioning of control electrodes close to the qubits, with the attendant fabrication difficulties and introduction of additional decoherence sources, is avoided. At least two ways of doing this have been suggested. The more conceptually straightforward way is to control optically the matrix element connecting a pair of qubits; for example, Piermarocchi *et al.* (2002) suggest exploiting the exchange interaction between electron spins localized on quantum dots, and delocalized excitons. The density of excitons could be controlled optically, but the most elegant approach is to detune the laser from the excitonic absorption so that the excitons are only produced in virtual intermediate processes; these virtual processes can only occur while the laser field is present. The free spatial propagation of the excitons in two dimensions through a quantum well would then allow a relatively long-range interaction between quantum dots.

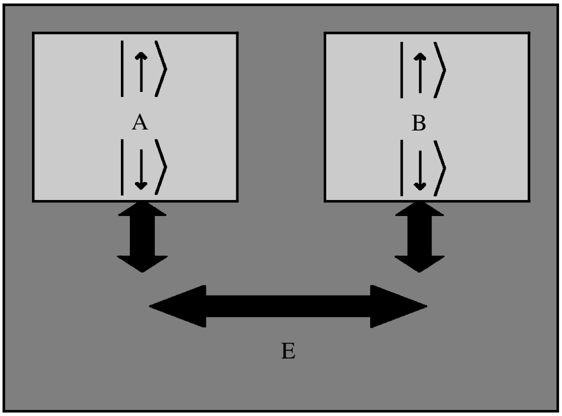

Figure 2. Sketch of the configuration for two spatially separated qubits described in the text. The systems A and B (shown as two-level spins) carry the quantum information but cannot communicate with one another directly—only via their mutual interaction with their environment, E.

A second approach is to exploit the exclusion principle. So-called 'Pauli blocking' of exciton formation in a quantum dot can occur if a carrier is already present in the relevant electron or hole bound states. For example, in a quantum dot containing a single excess electron, absorption of a σ^+ photon to form an exciton with electron angular momentum $m_J^{\mathrm{e}} = -1/2$ and hole angular momentum $m_J^{\mathrm{h}} = +3/2$ dots can occur only if the excess electron already has $m_J^{\mathrm{e}} = +1/2$. This provides a way to make the interactions between nearby dots (which depend on the presence or absence of excitons) depend on the spin states of the excess electrons. Pazy *et al.* (2003) suggest using this process to form two-qubit gates, where the qubits are the pre-existing electron spin states; however, their scheme requires individual optical addressability of the quantum dots, which might be difficult to realize in practice.

3. Fluctuations, entanglement, and decoherence: limits on the figure of merit for quantum gates

We now go on to consider what the fundamental limits on producing entanglement in solids might be. Manipulations of *single* qubits can be performed using entirely classical fields, for example, classical microwaves and a classical magnetic field in a conventional magnetic resonance experiment. Indeed, for such a single-qubit gate one generally *wants* the external field to be as classical as possible; an analysis of the way in which the single-qubit manipulation becomes ideal in the classical-field limit is given in van Enk & Kimble (2001). For entangling two-qubit gates, however, the situation is different; here we need an interaction between the two qubits which is sensitive to the state of each.

Let us suppose that two qubits, A and B, are spatially separated. Since all interactions in physics are ultimately *local*, the qubits can then only be coupled by their mutual interaction with some other, delocalized system (the environment, E, which could be the electromagnetic field or some other field supported by a solid environment, such as a Fermi sea of conduction electrons). A schematic of the situation is shown in figure 2. It is well known that the environment acts as a source of decoherence, but in this situation it also makes an essential contribution to the unitary evolution of the qubits, by transporting quantum information from one site to another. In mathematical terms, the evolution through a short time Δt of the reduced density matrix $\hat{\rho}_{\mathrm{AB}} = \mathrm{Tr}_{\mathrm{E}}[\hat{\rho}]$ of the qubits when they are coupled to the environment is often described by a Lindblad master equation of the form

$$\Delta\hat{\rho}_{\mathrm{AB}} = \Delta t\biggl\{-\mathrm{i}[\hat{H}_{\mathrm{eff}}, \hat{\rho}_{\mathrm{AB}}] + \sum_{\mu}(\hat{L}_{\mu}\hat{\rho}_{\mathrm{AB}}\hat{L}_{\mu}^{\dagger} - \tfrac{1}{2}\{\hat{L}_{\mu}^{\dagger}\hat{L}_{\mu}, \rho_{\mathrm{AB}}\})\biggr\}. \tag{3.1}$$

If we do this, the fact that the same environment causes both the coupling of the qubits and their decoherence means that the effective Hamiltonian H_{eff} and the Lindblad operators $\{L_{\mu}\}$ will not be independent. To put it another way, any two-body interaction between the qubits, and hence any entanglement, will come at the price of decoherence.

Let us see how this works out. First, we note that it is common, especially in quantum optics, to treat decoherence using a Markovian assumption (essentially, assuming that the environment instantly 'forgets' what has been done to it). It is important to realize that this is not fully consistent with the role of the environment in coupling the qubits: the environment has to 'remember' what was done to it by qubit A, in order that it can itself influence qubit B. We have to keep track of the environmental 'memory' at least for long enough for this process to occur. The results given below follow from doing just this, working to second order in the interaction between the qubits and the environment. We follow the analysis of the competition between entanglement and decoherence given in Fisher (2002); other treatments of the coupled dynamics of multi-qubit systems and their environment can be found in Zanardi (1998), Zanardi & Rossi (1998) and Lidar *et al.* (2001).

The key point is that the two-qubit (entangling) part of the effective interactions and the Lindblad operators are both determined solely by correlation functions of the environment. This result is not surprising when one considers the response of a system to a classical perturbation: suppose qubit A were a classical object and produced a perturbation on the environment proportional to some operator $\hat{O}_{\mathrm{A}}$; furthermore, suppose qubit B responded to a (different) environmental quantity $\hat{O}_{\mathrm{B}}$. How is the value of $\hat{O}_{\mathrm{B}}$ affected when qubit A is coupled into the environment at $t = 0$, say? In the limit of weak coupling, the answer is given by the well-known linear response theorem (Kubo 1957), which states that the time-dependent response of $\langle\hat{O}_{\mathrm{B}}\rangle$ to a stimulus $f(t)\hat{O}_{\mathrm{A}}$ is

$$\delta\langle\hat{O}_{\mathrm{B}}\rangle(t) = \int_0^t \phi_{\mathrm{BA}}(t-t')f(t')\,\mathrm{d}t' + \mathrm{O}(f^2). \tag{3.2}$$

The response function ϕ_{BA} is

$$\phi_{\mathrm{BA}}(\tau) = \mathrm{i}\langle[\hat{O}_{\mathrm{A}}(\tau), \hat{O}_{\mathrm{B}}(0)]\rangle = \mathrm{i}[C_{\mathrm{AB}}(\tau) - C_{\mathrm{BA}}(-\tau)], \tag{3.3}$$

where the correlation functions $C_{ab}(\tau)$ are

$$C_{ab}(\tau) \equiv \langle\hat{O}_a(\tau)\hat{O}_b(0)\rangle, \quad a, b \in \{\mathrm{A}, \mathrm{B}\}. \tag{3.4}$$

(All expectation values are taken in the *unperturbed* environment.) The causal Fourier transform of ϕ_{BA} is the dynamical susceptibility, which can be written (assuming the environment is in thermal equilibrium before the qubits are coupled to it) as

$$\chi_{\mathrm{BA}}(\omega) = \int_0^{\infty} \phi_{\mathrm{BA}}(\tau)\exp(\mathrm{i}\omega\tau)\,\mathrm{d}\tau = -\int_{-\infty}^{\infty} \mathrm{d}\omega'\,\frac{(1-\mathrm{e}^{-\beta\omega'})J_{\mathrm{BA}}(\omega')}{\omega-\omega'}, \tag{3.5}$$

where $\beta = 1/(k_B T)$. The quantities $J_{ab}(\omega)$ are just the Fourier transforms of the corresponding correlation functions,

$$J_{ab}(\omega) = \int_{-\infty}^{\infty} C_{ab}(\tau) \exp(i\omega\tau)\, d\tau, \quad a, b \in \{A, B\}. \tag{3.6}$$

For the 'classical' response, the causal structure of these relationships gives rise to dispersion relations connecting the real and imaginary parts of the response, and hence to a relationship between the imaginary (dissipative) part of the environment's response to the driving field and the joint fluctuation spectrum of the quantities O_A and O_B,

$$\mathrm{Im}(\chi_{AB}(\omega)) + \mathrm{Im}(\chi_{BA}(\omega)) = [J_{AB}(\omega)n(\omega) + J_{BA}(-\omega)n(-\omega)]. \tag{3.7}$$

Here $n(\omega) \equiv [\exp(\beta\omega) - 1]^{-1}$ is the Bose occupation number at frequency ω.

How does this change for the case where the environment is transmitting information between two *quantum* objects? Here we will write down the theory for the case in which the systems A and B are $S = 1/2$ spins, in the presence of a uniform field giving a spin-flip frequency ω_0 and coupled to the environment by an interaction

$$H_1 = \lambda(\hat{O}_A \hat{\sigma}_A^x + \hat{O}_B \hat{\sigma}_B^x). \tag{3.8}$$

We shall also assume that the spectral density of the bath is slowly varying in frequency; this will typically be the case if the environment is a bulk system. (This restriction can, however, be relaxed (Fisher 2002); this might be appropriate for cases such as qubits coupled by electromagnetic modes, or by electronic states of a small nanostructure.) In this case, a perturbative analysis (Fisher 2002) to second order in λ shows that the effective Hamiltonian appearing in equation (3.1) is $\hat{H}_{\mathrm{eff}} = \hat{H}_+ + \hat{H}_-$, where

$$\left.\begin{aligned}
\hat{H}_+ &= \lambda^2 \sum_{ab} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{J_{ab}(\omega)}{(\omega - \omega_0)} \hat{\sigma}_-^a \hat{\sigma}_+^b, \\
\hat{H}_- &= \lambda^2 \sum_{ab} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} J_{ab}(\omega) \frac{1}{(\omega + \omega_0)} \hat{\sigma}_+^a \hat{\sigma}_-^b \\
&= -\lambda^2 \sum_{ab} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{J_{ab}(\omega)}{(\omega - \omega_0)} e^{-\beta\omega} \hat{\sigma}_+^b \hat{\sigma}_-^a.
\end{aligned}\right\} \tag{3.9}$$

Provided the $J_{AB} \neq 0$ (i.e. provided that the fluctuations experienced by A and B are correlated), $\hat{H}_{\mathrm{eff}}$ contains terms such as $\sigma_+^A \sigma_-^B$, which can produce the desired entanglement between A and B. (This part of the Hamiltonian is rather like a two-centre generalization of the Lamb shift, and indeed is described as such by Lidar *et al*. (2001).) However, this entanglement comes at the cost of decoherence-inducing Lindblad operators $L_{\mu,+}$ and $L_{\mu,-}$, given by

$$\left.\begin{aligned}
\sum_\mu L_{\mu,+}^\dagger L_{\mu,+} &= \lambda^2 \sum_{ab} J_{ab}(\omega_0) \hat{\sigma}_-^a \hat{\sigma}_+^b, \\
\sum_\mu L_{\mu,-}^\dagger L_{\mu,-} &= \lambda^2 \sum_{ab} J_{ab}(-\omega_0) \hat{\sigma}_+^a \hat{\sigma}_-^b \\
&= e^{-\beta\omega_0} \lambda^2 \sum_{ab} J_{ab}(\omega_0) \hat{\sigma}_+^b \hat{\sigma}_-^a.
\end{aligned}\right\} \tag{3.10}$$

It is now clear that the coherent and incoherent parts of the evolution are related, since they depend on exactly the same correlation functions. The relationship is similar, but not identical, to that between the real (in-phase) and imaginary (out-of-phase) parts of the response in conventional linear-response theory. The reason for the difference is that we now have to distinguish between positive-frequency processes (in which the environment transiently absorbs energy from system A or B) and negative-frequency processes (in which the environment gives out energy). Both contribute (via the terms in H_+ and H_-, respectively) to the effective Hamiltonian (3.9), but each corresponds to a different type of decoherence (the Lindblad operators $L_{\mu,+}$ and $L_{\mu,-}$). There are therefore two separate relations that link the effective Hamiltonian when the system is operated at some given frequency ω to the positive- and negative-frequency incoherent terms at all other frequencies ω':

$$\hat{H}_s(\omega) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega'}{2\pi} \frac{[\sum_\mu L^\dagger_{\mu,s}(\omega') L_{\mu,s}(\omega')]}{\omega' - \omega}, \quad s \in \{+,-\}. \tag{3.11}$$

This relationship directly establishes the maximum figure of merit obtainable for a given gate, since it links the relative magnitudes of the coherent and incoherent terms in the evolution of the qubits. The following points are immediately apparent from equation (3.11).

(i) The relationship is independent of the coupling strength, λ, since both the coherent and incoherent terms are proportional to λ^2. So, as far as the figure of merit is concerned, there is no particular advantage in choosing qubits that are strongly or weakly coupled to the environment; a larger value of λ retains the advantage, however, that the time taken to perform each two-qubit operation is reduced.

(ii) The maximum figure of merit does, however, strongly depend on the form of the spectral functions $J_{ab}(\omega)$, and especially on the spectral weight for frequencies near the operating frequency, ω_0, relative to the total weight of the integrands in equation (3.9). This immediately suggests ways of engineering the environment to optimize the figure of merit.

(iii) One familiar example of this is in optics; in this case, the coupling to the electromagnetic field inevitably produces decoherence from spontaneous emission. The limits placed by spontaneous emission on QIP in ion traps have been studied previously (Plenio & Knight 1997). The decoherence constraints imposed by coupling to a free electromagnetic field are rather weak; this is essentially because the density of states at any normal operating frequency ω_0 is rather small in comparison with the total contributions to the integrals in equation (3.9).

(iv) The type of decoherence we describe here is 'irreducible', in the sense that it arises inevitably from the coupling between the qubits. A decoherence-free subspace (DFS) (see Lidar *et al.* 1998) cannot protect against it, since this would correspond to representing our quantum information entirely through states of the qubits that are annihilated by the Lindblad operators in equation (3.10); however, equation (3.11) tells us that this would also mean that these states were also annihilated by the effective Hamiltonian, H_{eff}. Hence, no two-qubit quantum operations would be possible.

(v) In any given realization, the dominant decoherence may or may not be of this 'irreducible' type. If the dominant decoherence comes from some other source, there may still be considerable value in using a DFS.

(vi) Finally, it should be noted that new effects may be expected in the strong-coupling limit, where the qubits are very strongly coupled to the environment and perturbation theory can no longer be used.

4. Conclusions and prospects

There is no shortage of exciting and innovative proposals for processing quantum information in solids. Hard facts are now needed about the feasibility of fabrication, gate-operation times, decoherence times and readout efficiencies of the candidate systems; a great deal of work is going into this area, and such data should be available within the next two to three years. The most pressing need is for an effective means of discriminating between proposals at an early stage and to pick out those most likely to meet the very demanding requirements for fault-tolerant quantum computation (Preskill 1998); it is hoped that the route outlined here to obtain limits on figures of merit for multi-qubit gates may play a role in this regard.

I thank Marshall Stoneham and Thornton Greenland for their support, collaboration, and interaction throughout the course of this work, and Tim Spiller, Bill Munro, Simon Benjamin and Martin Plenio for a number of further discussions. Some of this work was supported by the EPSRC under ROPA grant GR/M67865, and some through the IRC in Nanotechnology.

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